2018. M29

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2018

Mathematics

Paper 1

Higher Level

Friday, 8 June – Afternoon 2:00 to 4:30

300 marks

Examination number

Centre stamp

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Running total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A Concepts and Skills 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) Solve the simultaneous equations.

$$\begin{aligned} 2x + 3y - \ z &= -4 \\ 3x + 2y + 2z &= 14 \\ x \qquad\quad - 3z &= -13 \end{aligned}$$

(b) Solve the inequality $\frac{2x-3}{x+2} \geq 3$, where $x \in \mathbb{R}$ and $x \neq -2$.

Question 2 **(25 marks)**

(a) The first three terms of a geometric series are x^2, $5x - 8$, and $x + 8$, where $x \in \mathbb{R}$.
Use the common ratio to show that $x^3 - 17x^2 + 80x - 64 = 0$.

(b) If $f(x) = x^3 - 17x^2 + 80x - 64$, $x \in \mathbb{R}$, show that $f(1) = 0$, **and** find another value of x for which $f(x) = 0$.

Show:

Other value:

(c) In the case of one of the values of x from part **(b)**, the terms in part **(a)** will generate a geometric series with a finite sum to infinity.
Find this value of x and **hence** find the sum to infinity.

$x =$

$S_\infty =$

previous	page	running

Question 3 **(25 marks)**

(a) Let $h(x) = \cos(2x)$, where $x \in \mathbb{R}$.
A tangent is drawn to the graph of $h(x)$ at the point where $x = \frac{\pi}{3}$.
Find the angle that this tangent makes with the positive sense of the x-axis.

(b) Find the average value of $h(x)$ over the interval $0 \leq x \leq \frac{\pi}{4},\ x \in \mathbb{R}$.
Give your answer in terms of π.

Question 4 **(25 marks)**

(a) Prove, using induction, that if n is a positive integer then

$$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta), \text{ where } i^2 = -1.$$

(b) Hence, or otherwise, find $\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3$ in its simplest form.

previous	page	running

Question 5 **(25 marks)**

(a) The *Sieve of Sundaram* is an infinite table of arithmetic sequences.
The terms in the first 4 rows and the first 4 columns of the table are shown below.

4	7	10	13		
7	12	17	22		
10	17	24	31		
13	22	31	40		

(i) Find the **difference** between the **sums** of the first 45 terms in the first two rows.

(ii) Find the number which is in the 60^{th} row and 70^{th} column of the table.

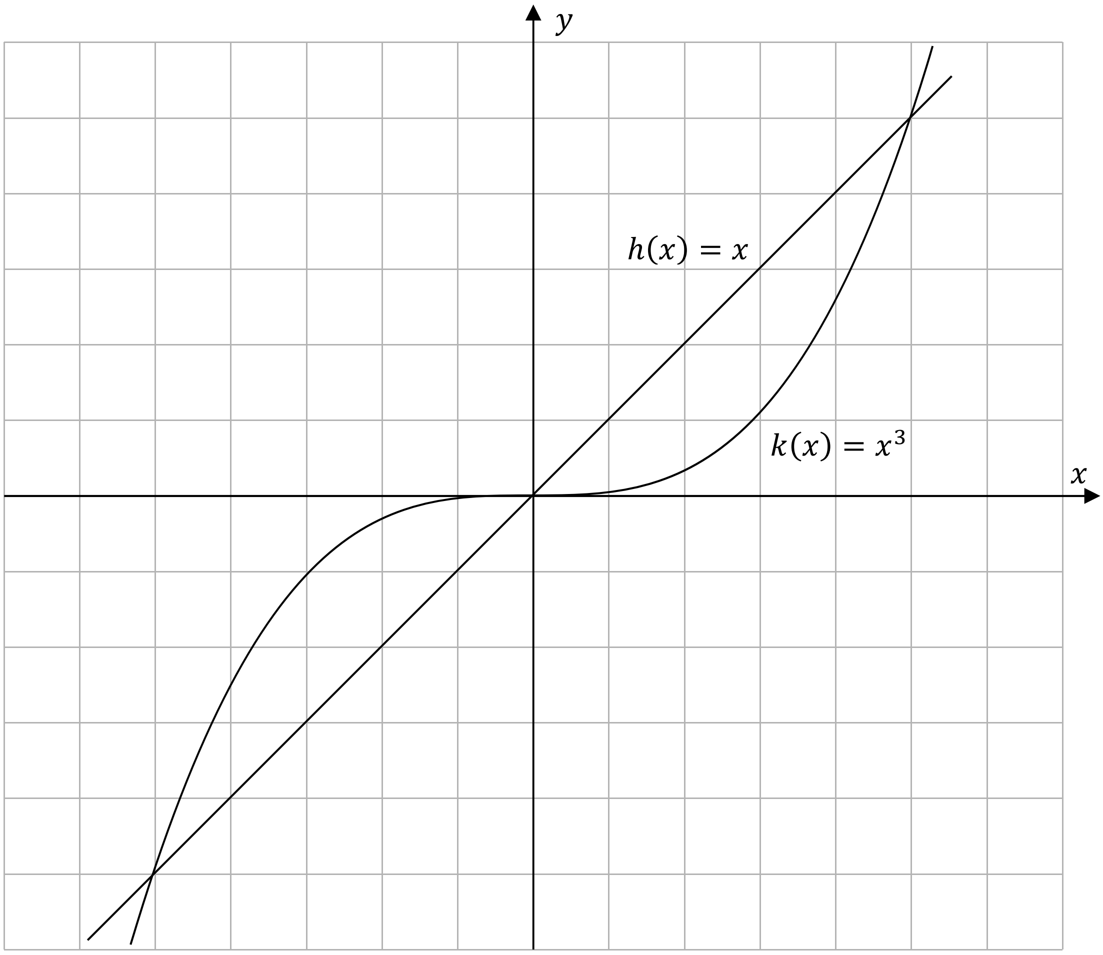
(b) The first two terms of a sequence are $a_1 = 4$ and $a_2 = 2$.
The general term is defined by $a_n = a_{n-1} - a_{n-2}$, when $n \geq 3$.
Write out the next 6 terms of the sequence **and hence** find the value of a_{2019}

$a_{2019} =$

previous	page	running

Question 6 **(25 marks)**

Parts of the graphs of the functions $h(x) = x$ and $k(x) = x^3$, $x \in \mathbb{R}$, are shown in the diagram below.



(a) Find the co-ordinates of the points of intersection of the graphs of the two functions.

(b) **(i)** Find the total area enclosed between the graphs of the two functions.

(ii) On the diagram on the previous page, using symmetry or otherwise, draw the graph of k^{-1}, the inverse function of k.

Section B	Contexts and Applications	150 marks

Answer **all three** questions from this section.

Question 7 **(55 marks)**

The time, in days of practice, it takes Jack to learn to type x words per minute (wpm) can be modelled by the function:

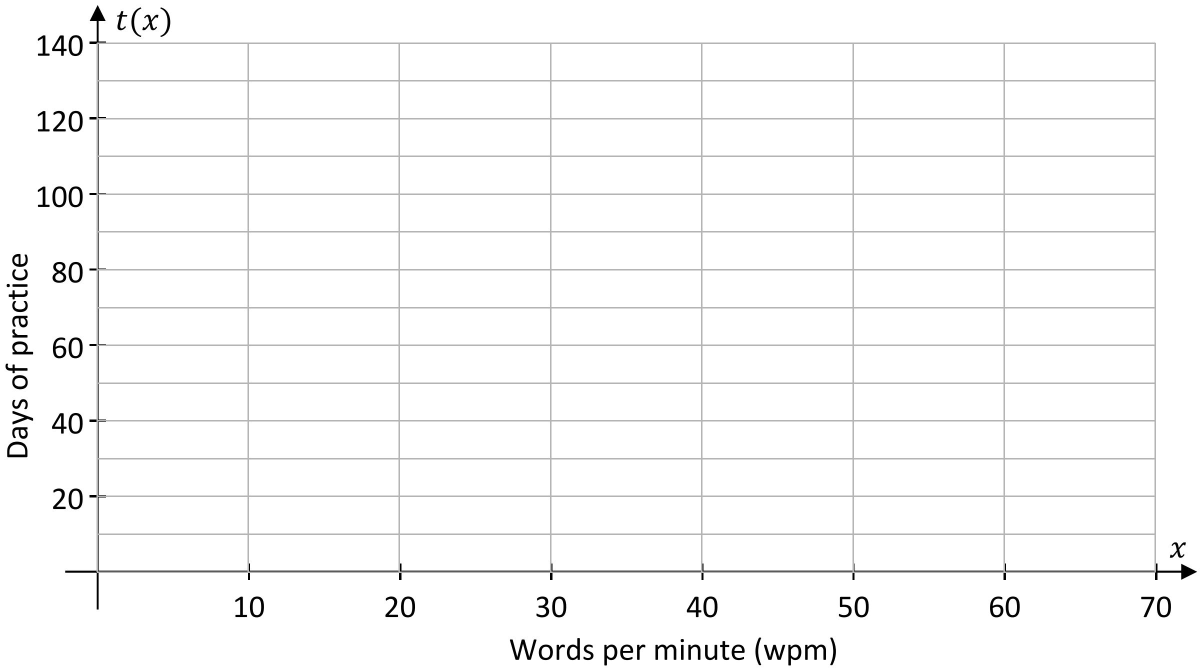
$$t(x) = k\left[\ln\left(1 - \frac{x}{80}\right)\right], \text{ where } 0 \leq x \leq 70,\ x \in \mathbb{R}, \text{ and } k \text{ is a constant.}$$

(a) Based on the function $t(x)$, Jack can learn to type 35 wpm in 35·96 days. Write the function above in terms of k and **hence** show that $k = -62{\cdot}5$, correct to 1 decimal place.

(b) Find the number of wpm that Jack can learn to type with 100 days of practice. Give your answer correct to the nearest whole number.

(c) Complete the table below, correct to the nearest whole number **and hence** draw the graph of $t(x)$ for $0 \leq x \leq 70,\ x \in \mathbb{R}$.

x (wpm)	0	10	20	30	40	50	60	70
$t(x)$ (days)								



(d) A simpler function that could also be used to model the number of days needed to attain x wpm is $p(x) = 1{\cdot}5x$.
Draw, on the diagram above, the graph of $p(x)$ for $0 \leq x \leq 70$, $x \in \mathbb{R}$.

(e) Let $h(x) = p(x) - t(x)$.

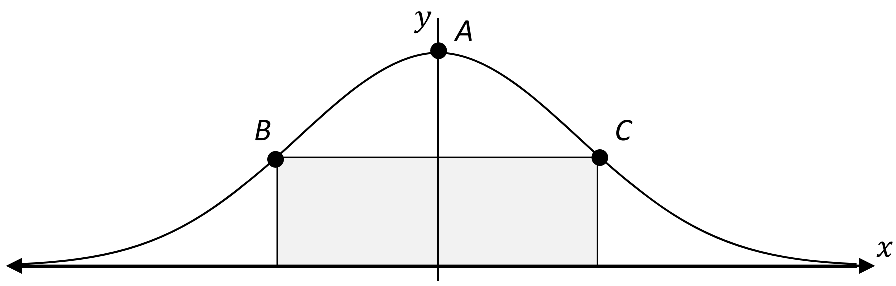
(i) Use your graphs above to estimate the solution to $h(x) = 0$ for $x > 0$.

(ii) Use calculus to find the maximum value of $h(x)$ for $0 \leq x \leq 70$, $x \in \mathbb{R}$.
Give your answer correct to the nearest whole number.

previous	page	running

Question 8 **(40 marks)**

The graph of the **symmetric** function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$ is shown below.



(a) Find the co-ordinates of *A*, the point where the graph intersects the y-axis.
Give your answer in terms of π.

(b) The co-ordinates of *B* are $\left(-1, \frac{1}{\sqrt{2\pi e}}\right)$. Find the area of the shaded rectangle in the diagram above. Give your answer correct to 3 decimal places.

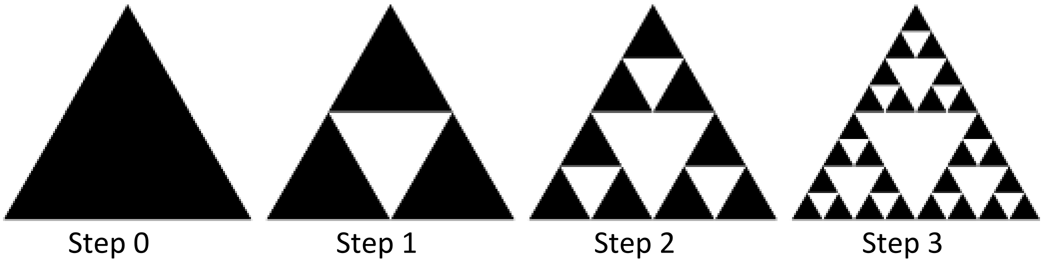
(c) Use calculus to show that $f(x)$ is decreasing at *C*.

(d) Show that the graph of $f(x)$ has a point of inflection at *B*.

previous	page	running

Question 9 **(55 marks)**

The diagram below shows the first 4 steps of an infinite pattern which creates the *Sierpinski Triangle*. The sequence begins with a black equilateral triangle. Each step is formed by **removing** an equilateral triangle from the centre of each black triangle in the previous step, as shown. Each equilateral triangle that is removed is formed by joining the midpoints of the sides of a black triangle from the previous step.



(a) The table below shows the number of black triangles at each of the first 4 steps **and** the fraction of the original triangle remaining at each step. Complete the table.

Step	0	1	2	3
Number of black triangles	1			
Fraction of the original triangle remaining	1		$\frac{9}{16}$	

(b) **(i)** Write an expression in terms of n for the number of black triangles in step n of the pattern.

(ii) Step *k* is the first step of the pattern in which the number of black triangles **exceeds** one thousand million (i.e. 1×10^9) for the first time. Find the value of *k*.

(c) **(i)** Step h is the first step of the pattern in which the fraction of the original triangle remaining is less than $\frac{1}{100}$ of the original triangle. Find the value of h.

(ii) What fraction of the original triangle remains after an infinite number of steps of the pattern?

This question continues on the next page

previous	page	running

(d) **(i)** The side length of the triangle in Step 0 is 1 unit. The table below shows the total **perimeter** of all the black triangles in each of the first 5 steps.
Complete the table below.

Step	0	1	2	3	4
Perimeter	3		$\frac{27}{4}$		

(ii) Find the total perimeter of the black triangles in step 35 of the pattern.
Give your answer correct to the nearest unit.

(iii) Use your answers to part **(c)(ii)** and part **(d)(ii)** to comment on the total **area** and the total **perimeter** of the black triangles in step n of the pattern, as n tends to infinity.

Area:

Perimeter:

You may use this page for extra work.
Label any extra work clearly with the question number and part.

previous	page	running

Leaving Certificate 2018 – Higher Level

Mathematics – Paper 1

Friday, 8 June
Afternoon, 2:00 to 4:30